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Educational opportunity in early and middle childhood: Variation by place and age

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ABSTRACT

I use standardized test scores from roughly 45 million students to describe the temporal structure of educational opportunity in over 11,000 school districts—almost every district in the US. For each school district, I construct two measures: the average academic performance of students in grade 3 and the within-cohort growth in test scores from grade 3 to 8. I argue that third grade average test scores can be thought of as measures of the average extent of educational opportunities available to students in a community prior to age 9. Growth rates in average scores from grade 3 to 8 can be thought of as reflecting educational opportunities available to children in a school district between the ages of 9 and 14.

I document considerable variation among school districts in both average third grade scores and test score growth rates. Importantly, the two measures are uncorrelated, indicating that the characteristics of communities that provide high levels of early childhood educational opportunity are not the same as those that provide high opportunities for growth from third to eighth grade. This suggests that the role of schools in shaping educational opportunity varies across school districts. Moreover, the variation among districts in the two temporal opportunity dimensions implies that strategies to improve educational opportunity may need to target different age groups in different places. One additional implication of the low correlation between growth rates and average third grade scores is that measures of average test scores are likely very poor measures of school effectiveness. The growth measure I construct does not isolate the contribution of schools to children's academic skills, but is likely closer to a measure of school effectiveness than are measures of average test scores.

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Abstract

I use standardized test scores from roughly 45 million students to describe the temporal structure of educational opportunity in over 11,000 school districts—almost every district in the US. For each school district, I construct two measures: the average academic performance of students in grade 3 and the within-cohort growth in test scores from grade 3 to 8. I argue that third grade average test scores can be thought of as measures of the average extent of educational opportunities available to students in a community prior to age 9. Growth rates in average scores from grade 3 to 8 can be thought of as reflecting educational opportunities available to children in a school district between the ages of 9 and 14.

I document considerable variation among school districts in both average third grade scores and test score growth rates. Importantly, the two measures are uncorrelated, indicating that the characteristics of communities that provide high levels of early childhood educational opportunity are not the same as those that provide high opportunities for growth from third to eighth grade. This suggests that the role of schools in shaping educational opportunity varies across school districts. Moreover, the variation among districts in the two temporal opportunity dimensions implies that strategies to improve educational opportunity may need to target different age groups in different places. One additional implication of the low correlation between growth rates and average third grade scores is that measures of average test scores are likely very poor measures of school effectiveness. The growth measure I construct does not isolate the contribution of schools to children's academic skills, but is likely closer to a measure of school effectiveness than are measures of average test scores.

Educational opportunity in early and middle childhood:

Variation by place and age

Are public schools in the US engines of mobility or agents of inequality? Can schools in low-income communities provide a pathway out of poverty, or are the constraints of poverty too great for schools to overcome? Such questions are at the heart of debates about the role of education in social mobility in the US. Despite decades of research, we still lack clear answers, however.

In this paper I provide new evidence to inform these debates. This new evidence suggests that there is no clear answer to the question in part because the role of schooling in shaping educational opportunity varies substantially across places. Early childhood conditions more important in some places; educational opportunities during the elementary and middle school years appear more important in others.

In the first part of the paper, I use standardized test scores from roughly 45 million students to construct measures of the temporal structure of educational opportunity in over 11,000 school districts—almost every district in the US. The data span the school years 2008-09 through 2014-15. For each school district, I construct two measures: the average academic performance of students in grade 3 and the within-cohort growth in test scores from grade 3 to 8. I argue that average test scores in a school district can be thought of as reflecting the average cumulative set of educational opportunities children in a community have had up to the time when they take a test.

Given this, the average scores in grade 3 can be thought of as measures of the average extent of "early educational opportunities" (reflecting opportunities from birth to age 9) available to children in a school district. Prior research suggests that these early opportunities are strongly related to the average socioeconomic resources available in children's families in the district. They may also depend on other characteristics of the community, including neighborhood conditions, the availability of high-quality child

care and pre-school programs, and the quality of schools in grades K-3.

The growth in average test scores from grades 3 to 8 can likewise be thought of as a measure of the average extent of "middle childhood educational opportunities" available to children in a school district while they are roughly age 9 to 14. Given the prominence of schooling in children's lives at these ages, these middle childhood opportunities may depend in large part on the quality of the local elementary and middle schools. They may also depend on average family resources, of course, as well as other local conditions, including neighborhood characteristics and the availability of afterschool programs.

Given these two measures, average scores in eighth grade are then understood to reflect the cumulative set of early and middle grade educational opportunities available to children in a school district. The decomposition of eighth grade average scores into the two components, reflecting early opportunity and middle grades opportunity, provides insight into the temporal structure of educational opportunity. The availability of these two measures for over 11,000 school districts yields unprecedented insight into the geographic and temporal structure of childhood educational opportunity in the U.S.

In the second part of the paper, I describe both the relationship between these two measures and their association with socioeconomic characteristics of school districts. I find that the two measures are largely uncorrelated; early and middle-grades opportunities appear to be distinct and separable dimensions of local educational opportunity structures. Among districts with a given level of average test scores in third grade, there is wide variation in growth in average scores from third to eighth grade. Moreover, although both dimensions of opportunity are positively associated with district socioeconomic conditions, the correlation is much weaker for the middle grades growth dimension. There are many low-income school districts with relatively high measures of growth and many affluent districts with relatively low growth. Finally, I also examine the temporal opportunity structure separately by race/ethnic group and for poor and non-poor students.

The descriptive evidence I present is relevant to several scholarly and policy discussions. First, it suggests that the role of schooling (and factors that shape children's academic progress during the years they are in school) in shaping educational opportunity (and perhaps social mobility) varies across school districts. The answer to the question of whether schools exacerbate or ameliorate socioeconomic inequality may be "it depends on where you are." Second, the variation among districts in the two temporal opportunity dimensions implies that strategies to improve educational opportunity may need to target different age groups in different places. Third, one implication of the low correlation between growth rates and average third grade scores is that measures of average test scores are likely very poor measures of school quality. The growth measure I construct does not isolate the contribution of schools to children's academic skills, but is likely closer to a measure of school effectiveness than are measures of average test scores.

Background

Educational outcomes vary widely by socioeconomic status and race/ethnicity in the US. Children in high-income families, and those whose parent or parents have college degrees, systematically score higher on standardized tests and are more likely to attend and graduate from college than lower-income students and students whose parents did not attend college. Similar disparities are evident between White and Asian students and African American, Hispanic, and Native American Students (Chetty et al. 2017; Reardon 2011; Reardon, Robinson-Cimpian and Weathers 2015; Sirin 2005; Ziol-Guest and Lee 2016). This inequality in average group outcomes is *prima facie* evidence of systematic between-group differences in opportunity. But disparities in outcomes alone do not indicate the ways in which opportunities differ, nor the developmental stage when they are most salient. In particular, they do not tell us to what extent schools—and inequalities in schools—are to blame for these patterns. Here I briefly discuss two strands of scholarship that are relevant to this question: debates about the role of schools in

shaping inequality, and evidence regarding place-based opportunity structures.

Schools as "the great equalizer" in the United States

The debate regarding schools' role in providing educational opportunity and facilitating social mobility has a long history, particularly among sociologists. Three dominant arguments shape the debate. One position holds that schools reduce inequality of opportunity. The stark inequality in children's family backgrounds creates large differences in children's opportunities to learn, but school environments—in this argument—are less unequal than children's home environments. Evidence for this view comes from research showing, for example, that racial or socioeconomic achievement gaps widen in the summer when children are not in school, but narrow (or at least do not grow) when children are in school (Alexander, Entwisle and Olson 2001; Alexander, Entwisle and Olson 2007; Downey and Condron 2016; Downey, Hippel and Broh 2004). This evidence is sensitive to the scale used to measure academic performance, however; not all studies show these same patterns (von Hippel, Workman and Downey 2017). Additional support for this argument comes from studies showing that poor children benefit more from expanded time in school (via universal preschool enrollment, universal kindergarten, full-day kindergarten, and extended school days) than do non-poor children (Raudenbush and Eschmann 2015).

A second position is that schools have relatively little effect on the inequality of educational outcomes; family background is a far stronger force than schooling. In this view, most educational inequality is produced early in children's lives and by differences in family resources. This was the conclusion of the 1966 Coleman Report, and was, to some extent, the argument of Jencks and his colleagues (Coleman et al. 1966; Jencks 1972). Additional evidence for this view comes from studies that find that socioeconomic or racial achievement gaps are large when children arrive in formal schooling in kindergarten, and do not change appreciably during the schooling years (Reardon 2011; Reardon, Robinson-Cimpian and Weathers 2015).

Related to this argument is extensive evidence documenting the developmental importance of early childhood experiences. Family income when children are young is particularly consequential (relative to family income when children are older) for children's educational development (Duncan and Brooks-Gunn 1997; Duncan, Brooks Gunn and Klebanov 1994). Early childhood interventions can have significant and lasting impacts on children's outcomes (Duncan and Magnuson 2016; Heckman, Pinto and Savelyev 2013). And conditional on income, where one lives as a young child appears to have more effect on college attendance and income in young adulthood than does where one lives as an adolescent (Chetty, Hendren and Katz 2015). The salience of early childhood experiences may mean that experiences during middle childhood and adolescence are relatively unimportant in comparison.

Counter to this argument, however, are case studies and evaluations showing that schooling interventions or policies can have significant effects on achievement gaps, at least in some schools or as a result of specific interventions (Abdulkadiroglu et al. 2011; Bloom and Unterman 2012; Dobbie, Fryer and Fryer Jr 2011). Lottery-based studies of charter schools, likewise, reveal considerable heterogeneity in both charter and traditional public schools' effectiveness (Center for Research on Education Outcomes (CREDO) 2015; Tuttle, Gleason and Clark 2012). This implies that malleable features of schools can have sizeable effects on students' academic performance.

The third view is that schools are powerful agents of inequality. In this view, not only can schools have sizeable effects on student achievement, but social policies and economic forces conspire to ensure that schools in high-poverty neighborhoods are systematically inferior to those in affluent communities. In this view, schools exacerbate social inequalities, in large part because society systematically invests little in poor children's schools. Evidence for this comes from studies showing that schools in low-income communities have less qualified teachers (Boyd et al. 2005; Lankford, Loeb and Wycoff 2002) and weaker curricula (Darling-Hammond 1998). An older strain of research argues that high-poverty schools have systematically fewer financial resources (see, for example, Kozol 1967; Kozol 1991), though in many—but

not all—states this is no longer true (at least in terms of average per-pupil financial resources) (Chingos and Blagg 2017). An alternate, neo-Marxist version of this argument holds that capitalism requires an unequal schooling system in order to prepare students of different class background for their future roles in a capitalist economy (Bowles and Gintis 1976).

Each of these arguments has both supporting and countervailing evidence. This is because there is some truth to each of them, and because the role of schooling varies across place.

Geographic variation in educational opportunity

Much of the discussion of the role of schools or the importance of early childhood is concerned primarily with the average patterns of educational opportunity available to different socioeconomic or demographic populations. But recent research demonstrates that educational opportunity also varies significantly by location, even conditional on family income. Children's educational outcomes (test scores, high school graduation rates, and college enrollment and attendance rates) vary widely across place in the U.S. Chetty, Hendren, Kline, and Saez (2014), using tax records of 12 million children born in the U.S. in the early 1980s, demonstrate that this variation is substantial, even conditional on family income. Among children born to families at the 25th percentile of the income distribution, for example, college enrollment rates range from less than 25 percent to more than 65 percent across the 709 commuting zones[1] they study. That is, educational opportunity is a function of *place* as well as a function of family resources.

This is consistent with research on neighborhood effects, which argues that neighborhood contexts play a role in shaping educational outcomes (Chetty, Hendren and Katz 2016; Harding 2003; Sampson, Sharkey and Raudenbush 2008; Wodtke, Harding and Elwert 2011). Much of this literature,

[1] Commuting zones are collections of counties similar to metropolitan areas but covering the entire U.S. The average commuting zone includes about 4 counties.

however, focuses on the effects of neighborhood economic conditions; research has been less successful at identifying the mechanisms through which neighborhood contexts and community institutions shape educational opportunity. Chetty et al (2014) note that upward economic mobility of children born to low-income families is lower in places with lower test scores and in more segregated places. Both of these are consistent with a story in which the quality of local schools shapes opportunities for mobility: in segregated areas, poor children are more concentrated in a subset of high-poverty schools; these schools may be lower in quality, leading to lower test scores, which reduce future educational opportunities and may be reflected in lower wages. But the evidence is far from definitive. Indeed, in another paper Chetty and colleagues show that children's neighborhood contexts when they are young are more influential than their neighborhood conditions after age 10, a finding that suggests schools may not play a central role in shaping mobility (Chetty, Hendren and Katz 2015).

In short, the evidence is increasingly clear that educational opportunity and social mobility vary spatially. Less clear, however, is the role of schooling in shaping those patterns. Local contexts shape academic skills and human capital, but how? In this paper, I provide evidence to help answer that question, by describing evidence of the timing of these effects. By measuring average academic skills at different ages in each school district, I provide information on how educational opportunity varies by age across communities.

Temporal patterns of educational opportunity

Suppose we characterized each community on two dimensions of opportunity: opportunities available to children in early childhood and opportunities available during their middle childhood. Early opportunities might depend on experiences that children have in their homes, in child care and in preschool. These will be strongly influenced by the average family resources in a community (income, social capital, educational attainment), but may also depend on neighborhood conditions and local

context. For example, two equally poor communities may differ in the extent to which children are exposed to lead paint or other environmental toxins. Two equally affluent communities may differ in the quality of available pre-school programs. Middle-childhood opportunities may depend substantially on children's schooling experiences and the quality of the local schools, but also may be shaped by family resources and neighborhood conditions, the availability of after school activities, neighborhood safety, and so on.

Given these two dimensions, consider five potential patterns of the distribution of educational opportunities among communities. Each of these five corresponds to a panel in Figure 1, and each is characterized by three features: the variance of early childhood opportunities, the variance of middle childhood opportunities, and the correlation between the two. The top part in Figure 1 illustrate patterns of early and middle childhood opportunities; the bottom part shows the corresponding stylized patterns of outcomes at the end of early and middle childhood that would result.

A. Early experiences largely shape outcomes. In this case, early childhood educational opportunities vary widely among communities, but middle-childhood opportunities are similar across places. This might occur if, for example, early opportunities are very dependent on private resources (parental income and investments of time and money in children's development) while middle-childhood opportunities are structured by public institutions (such as schools) that are much more equal in the opportunities they provide than are families. This pattern would be consistent with the view that schools are equalizing forces in society, at least in comparison to out of school experiences.

B. Middle childhood experiences largely shape outcomes. In this case, educational opportunities in early childhood are much less variable than in middle childhood. This might occur if school quality were highly variable, but preschool quality and parenting practices were not related to family resources. Such a scenario is admittedly not very likely given what we know about the world and

the substantial impact of family resources on early childhood opportunities and development (Duncan and Brooks-Gunn 1999; Phillips and Shonkoff 2000).

C. Both early and middle childhood opportunities vary considerably, and are positively correlated. Here, there is really only a single dimension of opportunity: communities where children have above-average early opportunities tend to be those where middle childhood opportunities are also high, and vice versa. This might occur if school quality were dependent on average family socioeconomic resources, for example, or if family resources continue to play a powerful role in children's educational development while they are in school. In this scenario, inequality of outcomes would grow from early to middle childhood.

D. Both early and middle childhood opportunities vary considerably, but are uncorrelated. In this case, the factors that shape early childhood opportunities (such as family resources, preschool quality, environmental hazards) are not the same as those that shape later opportunities (such as schools or after-school programs). As a result, there are some communities where both early and middle childhood opportunities are high, some where both are low, and some where one is high and the other low. The presence of two distinct temporal dimensions of opportunity would suggest that strategies for improving opportunity might need to be targeted by both age and place.

E. Both early and middle childhood opportunities vary considerably, and are negatively correlated. In this case, middle childhood experiences tend to be compensatory. Those communities that provide low opportunities early in childhood (because of, for example, low family resources or few or low quality preschools) do provide high opportunities later, and vice versa.

Figure 1 here

In the remainder of the paper, I construct a version of Figure 1 empirically. Specifically, I use aggregated test score data to construct two measures for each school district in the US: a measure of

average third grade test scores (which can be thought of as the result of educational opportunities prior to third grade); and a measure of average learning rates from grade three to eight (which can be thought of as the result of educational opportunities during late elementary and middle school). The underlying data represent virtually all U.S. third through eighth graders' scores on state accountability tests from 2009 to 2015. I use these data to construct measures of 1) average initial (third grade) test scores and 2) growth rates of average scores in each district. Essentially, I partition each district's average eighth grade scores into two components—initial third grade levels and growth from third-eighth grade. This partition provides information about the temporal structure of educational opportunity in each school district.

Data

The test score data I use come from the Stanford Education Data Archive (SEDA), which includes estimates of the average test scores—by school district, grade, year, subject, and race/ethnicity—of students in almost every public school district in the United States (Reardon et al. 2017a). These estimates are based on roughly 300 million state accountability test scores (taken by roughly 45 million students) on math and English Language Arts (ELA) tests in grades 3-8 in the years 2009-2015 in every public school district in the United States. Cells with fewer than 20 students are suppressed in public SEDA data.

The raw test score data used to construct the SEDA data come from the federal ED*Facts* data collection system, which were provided by the National Center for Education Statistics under a restricted data use license. The data include, for each public school in the United States, counts of students scoring in each of several academic proficiency levels (often labeled something like "Below Basic," "Basic," "Proficient," and "Advanced"). These counts are disaggregated by race/ethnicity, grade (grades 3-8), test subject (math and ELA), and year (school years 2008-09 through 2012-15).

In the SEDA data, school-level proficiency counts are used to estimate average scores in each

school district. Charter schools' test scores are included in the public school district in which they are formally chartered or, if not chartered by a district, in the district in which they are physically located. Thus, in this paper I conceptualize a "school district" as a geographic catchment area that includes students in all local charter schools as well as in traditional public schools. Virtual schools—online schools that do not enroll students from any well-defined geographic area—are dropped from the sample. Such schools enroll less than half of one percent of all students in the US.

The test scores in each state, grade, year, and subject are placed on a common scale, so that performance can be meaningfully compared across states, grades, and years. First, each state's test scores are linked to the math and reading scales of the National Assessment of Education Progress (NAEP). The NAEP scale is stable over time and is vertically linked from fourth to eighth grade; this allows comparison of test scores among districts in different states and within a district across grades or years. Second, the NAEP scale is transformed linearly to facilitate grade-level interpretations. In this new scale, the national average 4^{th} grade NAEP score in 2009 is anchored at 4; the national average 8^{th} grade NAEP score in 2013 is anchored at 8. A one unit different in scores is interpretable as the national average difference between students one grade level apart (for much more detail on the linking method and scale, see Reardon, Kalogrides and Ho 2017). Details on the source and construction of the estimates is available on the SEDA website.

Any description of test score growth or change is dependent on the test metric used. The NAEP scale (or the linear transformation of it we use) is useful because it was developed to allow comparisons over time, across states, and across grades. Nonetheless, it is not the only defensible scaling of test scores. Another potential metric is one in which test scores are standardized relative to the national student test score distribution within each grade. In this scale, the average test score in each grade is 0 and the standard deviation is fixed at 1 in each grade. This is useful for comparing the relative magnitude of differences in test scores in one grade to another grade, but it may distort information about relative

growth rates. If the variation in true skills grows over time, the standardized metric will necessarily compress that growth and bias it toward zero, inducing a negative correlation between initial status and growth. In this paper I use both the NAEP metric (rescaled to grade equivalent units) and a standardized metric, though I focus primarily on the vertically linked NAEP metric because it allows meaningful changes in variance across grades. I use the standardized metric as a sensitivity check.[2]

Estimating average test scores and growth in average test scores

In each school district there are as many as 84 grade-year-subject specific measures of average test scores (six grades, seven years, and two subjects). I use these 84 estimates to construct measures of the average performance of students in a given grade (pooling across years and subjects) and the within-cohort growth rate of average scores across grades (pooling across cohorts and subjects). This approach is conceptually similar to that used by Hanselman and Fiel (2017) in their study of test score growth rates among California schools.

First, I define a cohort of observations as the set of observations corresponding to sequential grades in sequential years. Therefore, for example, one cohort is composed of students in third grade in 2009, fourth grade in 2010, fifth grade in 2011, and so on, through 8th grade in 2014. The next cohort consists of those in third grade in 2010 (eighth grade in 2015), and so on. Formally, I define a cohort as the spring of the year in which a group of students would have been in kindergarten (so that $cohort = year - grade$); thus the "2005 cohort" describes students who were in kindergarten in spring of 2005 (and who therefore appear in the SEDA data from 4th grade in 2009 through 8th grade in 2013). There are 12 cohorts represented in the SEDA data, from the 2001 cohort (in 8th grade in 2009) through the 2012 cohort (in 3rd grade in 2015).

[2] Other scalings of the test metric are defensible, of course. The indeterminacy of test metrics poses a challenge to any analysis of growth rates (Bond and Lang 2013; Ho 2008; Ho 2009; Reardon 2008). See Appendix for more discussion of the sensitivity of the estimates to alternative test scalings.

Note that this definition of "cohort" does not necessarily correspond to a constant group of students. That is, the students in 8th grade in 2014 in district d are not the same set of students who were in 3rd grade in district d in 2009. Some students may have been retained in grade or skipped a grade; some students may have left the district; others may have moved in. Such in- and out-migration may add random or systematic noise to our estimates of average growth rates; we may underestimate growth in places where those who leave are disproportionately higher-achieving than those who move in. Conversely, we may overestimate growth in places with the opposite in- and out-migration patterns or with high retention rates. This is a limitation inherent in the raw ED*Facts* data, which do not include student longitudinal records. I discuss this limitation more below.

Let $\hat{\mu}_{dygb}$ and $\omega_{dygb} = se(\hat{\mu}_{dygb})$ indicate the estimated average test score and its standard error for students in district d in year y, grade g, and subject b. Let $grd \in (3,4,5,6,7,8)$ and $coh \in (2001, \ldots, 2012)$ be continuous measures of grade and cohort, and let $math \in (0,1)$ be a binary indicator variable denoting the subject of an observation. Using data from all districts, years, grades, and subjects, I fit versions of the following precision-weighted multilevel model:

$$
\begin{aligned}
\hat{\mu}_{dygb} &= \beta_{0d} + \beta_{1d}(grd_{dygb} - 3) + \beta_{2d}(coh_{dygb} - 2005) + \beta_{3d}(math_{dygb} - .5) + u_{dygb} + e_{dygb} \\
\beta_{0d} &= \gamma_{00} + \mathbf{X}_d\boldsymbol{\Gamma}_0 + v_{0d} \\
\beta_{1d} &= \gamma_{10} + \mathbf{X}_d\boldsymbol{\Gamma}_1 + v_{1d} \\
\beta_{2d} &= \gamma_{20} + \mathbf{X}_d\boldsymbol{\Gamma}_2 + v_{2d} \\
\beta_{3d} &= \gamma_{30} + \mathbf{X}_d\boldsymbol{\Gamma}_3 + v_{3d} \\
e_{dygb} &\sim N(0, \omega_{dygb}^2);\ u_{dygb} \sim N(0, \sigma^2);\ \begin{bmatrix} v_{0d} \\ v_{1d} \\ v_{2d} \\ v_{3d} \end{bmatrix} \sim MVN(0, \boldsymbol{\tau}^2),
\end{aligned}
\tag{1}
$$

I fit the these models via maximum likelihood, treating ω_{dygb}^2 as known (it is the square of the standard error of $\hat{\mu}_{dygb}$). The variance term σ^2 and the $\boldsymbol{\tau}^2$ matrix are estimated.

I first fit this model with no district-level covariates ($\mathbf{X}_d$). This model provides estimates of a number of parameters of interest: the average third-grade test score in each district d (β_{0d}), the average

within-cohort growth rate of test scores from grades 3 to 8 in district d (β_{1d}), the variances of these two parameters in the population of all districts, and the correlation between grade 3 average scores and growth rates. Given the framework above, we can think of β_{0d} as a measure of the average educational opportunities children in district d have prior to the end of grade 3. Likewise, we can think of β_{1d} as a measure of the average educational opportunities children have to learn the tested material between grades 3 and 8. The average test scores in district d in 8th grade are therefore the sum of average grade 3 scores and 5 years of growth: $\beta_{0d} + 5\beta_{1d}$.

Because $\hat{\mu}_{dygb}$ is scaled to have an average value of 4 among 4th graders in 2009 and an average of 8 among 8th graders in 2013, the coefficients β_{0d} and β_{1d} reflect grade-levels units. Note that $\beta_{0d} = 3$ implies that students in district d have the same average scores in 3rd grade as the average 2008 third grader in the US. Likewise, $\beta_{1d} = 1$ implies that students in district d have the same average learning rate from grade 3 to 8 as the average student in the US (in the 2005 cohort). A value of $\beta_{1d} = 1.1$ or $\beta_{1d} = 0.90$, for example, would imply that the performance of the average student in district d improves or declines, respectively, 10 percent (one-tenth of a grade-level per year) faster or slower, respectively, than the average public school student in the US from 3rd to 8th grade.

Of particular interest here is the joint distribution of β_{0d} and β_{1d}. This is given by $\left[\binom{\gamma_{00}}{\gamma_{10}}, \boldsymbol{\tau}^2_{[01]}\right]$, where $\boldsymbol{\tau}^2_{[01]} = \begin{bmatrix}\tau_{00} & \tau_{01}\\ \tau_{01} & \tau_{11}\end{bmatrix}$ is the 2-x-2 upper-left submatrix of $\boldsymbol{\tau}^2$. This joint distribution is our primary focus: τ_{00} and τ_{11} describe the variances of β_{0d} and β_{1d}, respectively, and their correlation is computed as $r_{01} = \tau_{01}(\tau_{00} \cdot \tau_{11})^{-1/2}$. Note that I estimate the covariance matrix $\boldsymbol{\tau}^2_{[01]}$ via maximum likelihood using the model above, rather than from the observed variances and covariance of the estimated (and therefore error-prone) β_{0d}'s and β_{1d}'s.

In addition to providing estimates of the parameters of the joint distribution of β_{0d} and β_{1d}, the model also provides estimates of β_{0d} and β_{1d} for each district. I use the Empirical Bayes (EB) "shrunken"

estimates of these parameters, denoted β_{0d}^{*} and β_{1d}^{*}. The model provides estimates of the reliability of each of these estimates; as well as a measure of their average reliability.

The other coefficients in the model are of less direct interest for our purposes here. β_{2d} indicates the average within-grade (cohort-to-cohort) change per year in average test scores in district d; and β_{3d} indicates the average (within grade and year) difference in math and reading scores in district d.

To estimate the association between district characteristics (denoted by the vector $\mathbf{X}_d$) and average test scores (β_{0d}) and test score growth (β_{1d}), I fit models that add $\mathbf{X}_d$ as predictors of the district parameters in Model (1) above.

Measuring average socioeconomic status among students enrolled in a school district

In order to measure the socioeconomic characteristics of the families of children, I use data from the American Community Survey (ACS). The ACS includes detailed socio-demographic data for families living in each school district in the U.S.; these tabulations are available through the School District Demographic System (SDDS). I use data from the 2006-10 SDDS tabulations because they include tabulations of family characteristics among families with school-age children enrolled in public schools.

In particular, I use six measures of the socioeconomic composition of families living in a district with children enrolled in public schools: 1) median family income; 2) percent of adults with a bachelor's degree or higher degree; 3) poverty rate; 4) unemployment rate; 5) Supplemental Nutritional Assistance Program (SNAP) eligibility rate; and 6) the percent of families headed by a single mother. Each of these is available separately by race/ethnicity (for racial/ethnic groups of sufficient local population size).

I construct a measure of each district's average socioeconomic status as the first principal component of the six measures above. This measure is standardized to have a mean of zero and a standard deviation of 1. To give a sense of how this measure is scaled, Table 1 describes the average characteristics of school districts at various values of the SES composite.

Table 1 here

Analytic sample

The data I use here include 11,315 school districts for which I am able to compute a socioeconomic status variable and for which the SEDA data include measures of academic achievement. Districts not included in the sample are predominantly very small districts for which samples are too small for SDDS to report socioeconomic characteristics or that have fewer than 20 students total per grade (in which case the SEDA data do not include estimates of average test scores). There are 824 districts for which the ACS SES variable cannot be constructed; these are small districts (averaging 43 students per grade) and contain fewer than 1 percent of US public school students. The 11,315 districts in the analytic sample collectively enroll roughly 3.7 million students per grade (roughly 99 percent of all public school students in the U.S.).

How do grade three average scores and growth rates vary among districts?

Model 1 provides estimates of the average grade 3 test scores and the average grade 3-8 growth rate in each district. Importantly, it also provides maximum likelihood estimates of the variances and correlation of these parameters. Recall that we can think of the grade three test score average as a measure of "early educational opportunities" in a district; the growth rate serves as a proxy for "growth opportunities"—the extent of educational opportunities in grades three to eight.

Table 2 presents the parameters describing the joint distribution these two measures. The left panel reports the results based on the preferred grade-equivalent NAEP scale; the right panel reports comparable results based on the standardized scale. Each panel includes a column for math and ELA scores, as a well as results from the model that pools the data and estimates a common grade 3 level and growth rate for both subjects.

In the average school district, third grade average test scores are roughly one-sixth of a grade level above the national average, and increase by 0.97 grade levels per grade.[3] By third grade, test scores vary substantially across school districts. The standard deviation of district average third grade scores is almost one grade level (0.98 grade levels), meaning that roughly one third of school districts have average third grade test scores more than one grade level above or below the national average (one sixth above and one sixth below).

Table 2 here

Perhaps surprisingly, there is only a very weak—and negative—correlation between average third grade scores and growth rates ($r = -0.13$). This means that knowing a district's average third grade scores tells us almost nothing about the rate at which average scores change from third to eighth grade. Or, put in terms of opportunity structure, the communities where children experience high opportunities to learn in early childhood and early elementary school are not necessarily those where opportunities to learn are high in the elementary and middle school years, and vice versa.[4]

Although there is a weak and negative correlation between grade 3 levels and growth rates, that does not imply that there is no association between 8th grade scores and growth rates. Since average 8th grade scores are in part the result of growth rates, we would expect them to be positively correlated, and they are, though the correlation is moderate ($r = 0.49$). This suggests that 8th grade average scores carry more signal regarding growth rates than do 3rd grade scores. However, if we estimate the correlation between growth rates and average scores across all grades 3-8 (which is more typical of the level of detail

[3] Note that there are three reasons that the average district's scores are not equal to the national average. First, more small districts have above average test scores and slightly lower than average growth rates, so the unweighted averages across districts are not identical to the enrollment-weighted averages. Second, some very small districts are not included in the analytic sample of 11,315 districts. Third, the national average is constructed relative to students in the 2005 cohort (grade 4 in 2009, grade 8 in 2013), but districts' average scores are computed for all cohorts in the SEDA data (cohorts 2001-2012). The average third grade scores over all cohorts were slightly higher than those in the 2005 cohort, while the average growth rate over all cohorts was somewhat lower than that of the 2005 cohort.

[4] As noted above, this correlation is sensitive to the scale used to measure test scores.

publicly available about schools), the correlation is small ($r = 0.21$).

The righthand panel of Table 2 repeats the analysis using the standardized test score scale. In this scale, the correlation between growth rates and grade 3 average scores is similar, but slightly more negative than the estimate based on the grade-equivalent NAEP scaled scores. Again, average opportunities prior to third grade are a very poor predictor of average growth rates.

One additional feature of Table 2 is worth noting. The second and third columns of each panel show the estimate separately for math and reading tests. There is much more between-district variation in growth rates in math scores than in reading (the SD of growth rates is 40 percent larger in math than in reading), and – at least in the NAEP scale results – there is much less between-district variation in third grade achievement in math than in reading (the SD is 15 percent smaller in math than reading). This is consistent with the commonly-held belief that math skills are more affected by schooling, while reading skills are affected by both home and school environments. Early childhood and early elementary opportunities to learn to read may be more variable than opportunities to learn math skills, but growth in math scores from grade 3 to 8 appears to vary much more than growth in reading scores. Moreover, the correlation of growth and 8th grade scores is much higher for math than for reading ($r = 0.69$ for math versus $r = 0.21$ for reading). In other words, 8th grade math scores are a reasonably good proxy for growth rates in math, potentially because students' math skills (particularly those measured by standardized math tests) are shaped largely by opportunities to learn during the elementary and middle school years.

That said, in the interest of parsimony, I focus for the remainder of this paper on models that pool the estimates across math and reading. Given the relatively high within-district correlations between math and reading grade 3 scores ($r = 0.90$) and between math and reading growth rates ($r = 0.66$), models that pool the results across subjects capture most of the relevant information. Moreover, while growth rates and grade three levels are estimated reliably in all of the models here (generally above

0.75), they are lower in the subject-specific models than the pooled models (where the grade three averages are estimated with reliability 0.96 and the growth rates are estimated with reliability 0.86). The higher precision of the pooled models allows for sharper distinctions among districts. While there may indeed be important differences in that factors that shape opportunities for math and reading skill development, those issues are outside the scope of my analysis here.

How large is the variation in growth rates?

It is clear from Table 2 that average test scores in grade 3 are uninformative as predictors of growth rates, perhaps that is because there is relatively little variation in growth rates. It is useful therefore to quantify the magnitude of the variation in growth rates. The standard deviation of growth rates is 0.135 grade levels/year, or equivalently, 0.675 grade levels from grade 3 to 8. This means that in roughly one-sixth of districts test scores improve by two-thirds or more of a grade level from grades 3 to 8; in another one-sixth of districts scores fall behind by two-thirds or more of a grade level. Another way to quantify this is to note that a growth rate of 1.135 indicates that students' scores increase 13.5 percent faster than the national average (a note that increase of 13.5 percent of a school year is roughly an additional 25 school days/year in the typical district, not a trivial amount). So there is considerable variation among school districts in average growth rates.

Another way to quantify the relative magnitude of the variation in district test score growth rates is to compare the magnitude of between-district variation in growth rates to the magnitude of between-district variation in grade 3 test scores. Consider two school districts, one in which students' third grade scores are at the national average but growth rates are one standard deviation above the national average; and one in which students' third grade scores are one standard deviation above the national average but growth rates are at the national average. In which district are students' scores higher by eighth grade, and by how much? These calculations are shown in the bottom panel of Table 2.

A standard deviation difference in growth rates experienced over 5 years from grade 3 to 8 is equivalent to a 70 percent of a district standard deviation in grade three levels. That is, in 5 years, students in the average-early-opportunity/high-growth-opportunity district make up 70 percent of the grade three gap relative to a high-early-opportunity/average-growth-opportunity district. These results hold in both the reported scales.

Where in the US are growth opportunities highest?

Figures 2 and 3 display the geographic patterns of grade 3 average scores and grade 3-8 growth rates. Figure 2 shows that opportunities prior to grade 3 are highest in many of suburban and exurban school districts around metropolitan areas, particularly in the northeast, Midwest, and the California coast, and are low in much of the Deep South and the rural West. Growth opportunities in contrast are more varied. Tennessee is characterized by moderately low third grade scores but above average growth rates; Florida, in contrast, is characterized by slightly above average scores in grade 3 but very low average growth.

Figures 2 and 3 here

Both Table 2 and Figures 2 and 3 indicate both that there is considerable variation in both grade 3 average scores and growth rates, but that they are not highly correlated. This is more explicitly evident in Figure 4, which plots each district's estimated growth rate (on the vertical axis) against its grade 3-8 growth rate. The plot uses the EB estimates $\hat{\beta}^*_{0d}$ and $\hat{\beta}^*_{1d}$; imprecisely estimated values are shrunken toward the overall mean. Note that district estimates with a reliability less than 0.7 are not included in this or other figures (though their data are included in fitting Model 1).

Figure 4 here

Figure 4 makes clear that there is very little relationship between average third grade test scores and average growth. The figure can be divided into 4 quadrants defined by districts' early educational

opportunity and growth opportunities. In the upper right are districts characterized by high early educational opportunity and high growth opportunity; these are districts where students have high average achievement in grade 3 and have above average growth rates following that. In the lower left are districts characterized by the opposite pattern: low early and low-growth opportunity. The off-diagonal quadrants have high early/low growth and low early/high growth opportunity structures, respectively.

The striking feature of Figure 4 is the absence of a correlation between growth and initial scores. Among districts with high grade 3 scores there are many with high growth and many with low growth; the same is true among those with low initial scores. This suggests there is no significant floor or ceiling effect in the estimates (which is not surprising given that the data points reflect district average scores not individual student scores). Even among school districts with very high scores in third grade (three grade levels above average), some districts have very high growth; the same is true among initially low-performing districts.

Another perspective on Figure 4 is provided by considering districts with the same 8th grade average scores. The lack of a substantial correlation between growth and grade 3 scores implies that, among districts with the same 8th average grade scores, some have higher grade 3 scores and lower growth while others have lower initial status and higher growth. Figure 5 illustrates this: the plot is the same as Figure 4, but with lines representing levels of grade 8 average achievement drawn as isobars on the plot. Districts that fall anywhere on an isobar have the same average 8th grade achievement, despite differences in initial status and growth rates. For example, a district where initial scores are one grade level below and the average growth rates is 1.2 will have the same average 8th grade scores as one where initial scores are one grade level above average but growth rates are 0.8 (both districts will fall on the g8=8 line).

Figure 5 here

Chicago, for example (see Figure 6), has average 3rd grade test scores well below the national

average (about 1.4 grade levels below), but very high growth rates. New York City students have both average third grade scores and average growth rates. And in Henrico County, VA (suburban Richmond), third grade test scores are very high but growth rates are very low. As a result, 8th grade scores in Chicago, New York, and Henrico County are very similar (within a half grade level of each other) despite a range of 2.5 grade levels difference in their 3rd grade scores. Likewise, Detroit and Baltimore 8th grade test scores are very similar to one another (and very low, over 2.5 grade levels below the national average), but in Baltimore the low 8th grade scores are more the result of very low growth opportunities than low early opportunities, the opposite of Detroit.

Figure 6 here

Figure 6 highlights the 100 largest school districts in the U.S. The substantial variation among them on both the early and growth opportunity dimensions suggests that the variation evident in Figures 3-5 is not simply the result of idiosyncratic variation among small school districts or sampling noise. Each of these districts' estimates are based on hundreds of thousands or millions of test scores (Chicago's estimate is based on over 2 million test scores, for example).

How is average test score growth related to district socioeconomic status?

Figure 7 displays the association between the socioeconomic status measure and both grade 3 average scores (upper figure) and growth rates (lower figure). The fitted lines are estimated from a version of Model 1 that includes a cubic function of socioeconomic status (SES) as a predictor of each of the four distrct-level parameters in the model. SES is positively associated with grade three scores and growth rates, but the association is much stronger with grade 3 average scores ($r = 0.68$) than with growth rates ($r = .32$). These associations are shown graphically in Figure 7.

Figure 7 here

It may seem strange that both grade 3 average scores and growth rates are higher, on average, in

high-SES districts than in low-SES districts, but grade 3 average scores and growth are slightly negatively correlated. Figure 8 helps to clarify the patterns these patterns. Each panel of the figure highlights districts in a given SES quartile. Low-SES districts have generally, but not always, low average third grade scores, and many have lower than average growth rates. High-SES districts, in contrast, generally have above average grade 3 scores, but have above average growth rates only slightly more often than below average growth rates. In sum, socioeconomic status distinguishes where districts fall on the x-axis of Figure 8, but is not very predictive of where districts fall on the y-axis.

Figure 8 here

How do growth rates vary by student poverty status, race/ethnicity, and gender?

The analyses above demonstrate considerable variation among school districts in both early educational opportunities (as measured by average third grade test scores) and in growth rates from grade 3 to 8. How do these patterns differ by students' poverty status, race/ethnicity, and gender? Figure 9 displays average third grade test scores and growth rates for poor and non-poor students.[5] The left figure compares the average grade 3 scores of poor and non-poor students. On average, poor students' average third grade scores are 1.5 grade levels below those of their non-poor peers in the same district. Moreover, although there is considerable variation in the gap in average third grade scores between poor and non-poor students, almost every district in the US falls well below the 45-degree line in the figure. Of the roughly 10,000 school districts for which we have sufficient data to estimate average achievement levels by poverty status, in only a handful do poor and non-poor students arrive in third grade with equal academic skills (and in most of those few cases, both poor and non-poor students have low third grade scores).

[5] States report test scores by students' "economic disadvantage" status; each state can define "economic disadvantage" differently, though in practice, most use eligibility for free or reduced-price lunch to define economic disadvantage.

Figure 9 here

The right figure shows that the pattern is very different when comparing poor and non-poor students' growth rates. In most school districts, poor students' growth rates are very similar to those of non-poor students in the same district (most of the districts fall near the 45 degree line). The average within-district difference in growth rates between non-poor and poor students is 0.04 grade levels/year. That is, in the average district, poor students have third grade scores roughly 1.5 grade levels below their non-poor peers, and fall behind by an additional 0.2 grade levels by eighth grade. The difference in the early (pre-grade 3) opportunities of poor and non-poor students is much larger than the average difference in opportunities to learn in grades three to eight.

Table 3 reports the joint distributions of districts' grade 3 average test scores and growth rates by subgroup. Each column describes the distributions for a different group—by poverty status, race/ethnicity, and gender. The top row reveals the large differences in early educational opportunity by poverty status and race/ethnicity: poor students' average scores are 1.5 grade levels below non-poor students' in third grade. The racial/ethnic disparities are similarly large: the white-black and white-Hispanic gap are also roughly 1.5 grade levels in third grade.

Table 3 here

The second panel reports average growth rates. The average growth rate of poor students in the average district is 0.04 grade levels per year lower than that of non-poor students. The white-black difference in growth rates is -.055. These are meaningfully large, but not enormous, differences; they imply that the poor-nonpoor and white-black gaps grow by roughly 0.20 to 0.25 grade levels between third and eighth grade, a modest increase relative to the size of the gaps in third grade.[6] The Hispanic

[6] Hanselman and Fiel (2017) conduct a related but different analysis. Using 1998-2002 test score data from California, they find that black, Hispanic, and Asian students attend schools where, on average, the overall average growth rates are only slightly lower than in the schools attended by White students. Their analysis does not, however, identify race/ethnic-group specific growth rates, so is not directly comparable to the analyses here.

average growth rate is actually slightly higher than the white growth rate, meaning that white-Hispanic gaps narrow very slightly (by about one-eighth of a grade) between third and eighth grade. The Asian average growth rates are substantially higher, on average, than any other group, almost 0.15 grade levels/year higher than white growth rates. In the average district, Asian students have average scores roughly 0.7 grade levels higher than white students in grade three. This gap doubles, on average, by eighth grade.[7]

The last two columns report growth rates by gender. Girls have, on average, both higher third grade scores and higher growth rates than boys. By eighth grade, girls' average scores are roughly half a grade level higher than boys. Other research indicates that this difference is primarily due to the fact that girls substantially outperform boys on ELA tests, by nearly a grade level in eighth grade (Reardon et al. 2018).

Figure 10 summarizes the joint distribution of average third grade scores and growth rates for each subgroup (the gender figures are not shown since the male and female patterns differ relatively little from one another in comparison to the race/ethnic and socioeconomic differences). In most school districts, poor students, black students, and Hispanic students all have below average test scores in third grade, while nonpoor, white, and Asian students more commonly have above average scores. The growth rate patterns differ somewhat. Black students, for example, are generally in districts where both their early opportunities and growth opportunities generally low (in the lower left quadrant). The pattern is not so pronounced for Hispanic students and poor students: in many districts they have above average

[7] It is important to note that the average test score growth rates by subgroup are each estimated on a different sample of districts—those enrolling at least 20 students of that subgroup per grade. Therefore, the differences between subgroups' estimated average growth rates in Table 3 are not exactly the same as the average within-district average growth difference. One should read the differences in average growth rates here as suggestive of how achievement gaps change from the third to eighth grade, but not definitive. A better description of how gaps change (and how those rates of change are related to the magnitude of the gaps in third grade) could be obtained by limiting the analyses to a subset of districts with sufficiently large populations of the two subgroups of interest, and then estimating the average rate of change of within-district achievement gaps in this sample of districts. That analysis is beyond the scope of this paper.

growth rates, despite low average third grade scores. More generally, Figure 10 makes clear that patterns of both early opportunity and growth opportunity vary substantially by poverty status and race, but that growth opportunities are sometimes quite high for poor and Hispanic students.

Figure 10 here

Discussion, Part One: The Potential and Limits of Administrative Education Data

The data I use here, like most "administrative data," are the residuum of a set of federal and state educational bureaucratic processes. That is a fancy way of saying that the data—measures of student academic performance—were not designed and collected with social science research needs in mind. Each state tests all students in grades three through eight, and reports their scores—in aggregated and coarsened form—to the US Department of Education through the ED*Facts* system because federal law requires it. As a result, the data have both advantages and limitations.

Perhaps the most significant feature of the ED*Facts* data I use here is their population coverage; the data are based on the test scores of the full population of public school students in grades three to eight in each year from 2008-09 through 2014-15 (with some missing data as noted above). There are roughly 22 million third through eighth graders enrolled in public school each year in the US; each takes both a math and ELA test. Over the seven years of data I use, therefore, states administered roughly 300 million tests to these students. This is over 100 times as many tests as administered by NAEP over the same time period (NAEP administered roughly 600 thousand math and reading tests in grades 4 and 8 in each of the years 2009, 2011, 2013, and 2015). Even a school or district with only 25 students per grade would be represented by over 2000 test scores (7 years x 6 grades x 2 subjects x 25 students = 2,100 tests) in the ED*Facts* data, compared to only roughly 16 in the NAEP data. The ED*Facts* data therefore can provide a very high-resolution description of test score patterns even in very small schools or school districts.

The full population coverage of the ED*Facts* data make it possible to identify both general patterns of academic performance (such as the magnitude of achievement gaps) and heterogeneity in these patterns among subgroups, schools, districts, grades, and years. Sample-based analyses (even large samples like NAEP) might be able to provide reliable estimates of average test scores and growth rates for the nation as a whole, and by subgroup, or even by state (as is possible with NAEP data), are generally insufficient to describe the heterogeneity of these patterns across smaller geographic or organizational units, like school districts. As the analyses above show, there is a great deal of heterogeneity in these patterns among school districts. Moreover, not only do these data allow us to quantify the variation among school districts in the key parameters of interest here, but they would also enable us to identify interesting cases or sets of cases to study further. For example, we might be interested in what community and school characteristics foster high test score growth rates for poor students. We could identify a set of school districts in which poor students' growth rates are high, and then collect additional data, through case studies, about these districts; such case studies might be used to generate causal hypotheses that could be systematically tested in a larger set of districts. In other words, population-level data such as these

That said, the ED*Facts* data are far from ideal in a number of ways. First, the test scores are based on tests that differ across states and grades, and sometimes across years, making them not readily comparable except within a given state-grade-year. Second, the test scores are coarsened; scores are reported in broad categories that are given labels like "basic," "proficient," and "advanced." Not only does the coarsening destroy some information, but the categories are not defined in comparable ways across states, grades, and year. Third, the ED*Facts* data are reported in aggregated form, as counts of students in a given subgroup, school, grade, and year who score in each of 2-5 ordered performance categories; the ED*Facts* data does not include individual student records. This has two drawbacks: a) it is not possible to link students' scores longitudinally (or across subjects in the same grade and year); and b)

no data on individual student characteristics are included in the data. The latter means that we can only tabulate the test scores according to the subgroups reported in the data (which are those that states are required to report by law: race/ethnicity, gender, economic disadvantage, etc); we cannot construct student-level cross-tabulations (race-by-gender, for example).

These limitations are not trivial. The comparability issues due to differences in states and the definition of coarsened performance categories would seem to damn any attempt to compare performance except within individual state-grade-year-subjects. And the coarsening of the data would seem to muddy any statistical comparisons between the test score distributions in different districts, even in the same state-grade-year-subject, because the means and variances of each district's score distribution are not reported. My colleagues and I, however, demonstrate that it is possible to recover reliably-estimated test score means and variances in each district-grade-year-subject, and then to link these to a common national scale that enables meaningful comparisons across all districts in the US, and across grades and years (Reardon, Kalogrides and Ho 2017; Reardon et al. 2017b). Using these methods, we constructed the estimated district-specific test score means I use in this paper. These estimates are publicly available through the Stanford Education Data Archive (SEDA; at http:\\seda.stanford.edu).

One additional hurdle constrains the usefulness of the ED*Facts* data for research purposes. The raw ED*Facts* data are not publicly available; they require researchers obtain a restricted data-use license from the National Center for Education Statistics. Moreover, researchers are required to send all analyses to NCES for review before dissemination or publication, in order to avoid disclosure of individually identifiable information. The raw ED*Facts* data are unsuppressed, meaning that even if there is a single student of a given subgroup in a particular school-grade-year, that student's test score is reported in the raw ED*Facts* data files. NCES reviews research findings prior to dissemination in order to ensure that no individually identifiable information is released publicly. In order to enable us to make the estimated test score distributions publicly available through SEDA, NCES and ED*Facts* provided my colleagues and me

with a blanket disclosure agreement. Under this agreement, we suppress any estimate based on a cell size of less than 20 test scores. In addition, we add a small amount of random noise to all reported estimates in order to ensure that the estimation algorithm cannot be reverse-engineered to recover the underlying cell counts. With these provisos in place, NCES allows us to release our estimates publicly without further disclosure review. Because of this agreement, we are able to publicly disseminate estimates of the distributions of test scores in grades 3-8 from 2009-2015, all measured on a common scale, in virtually every school district in the US. These data are available at seda.stanford.edu.

Despite the value of SEDA, the available data cannot, however, overcome the limitations caused by the lack of student-level longitudinal data. Such data do, of course, exist. Most states now have education data systems that track individual students over time, so long as they remain in the state's public education system. One could, in theory, use states' student-level longitudinal data files (and the continuous, un-coarsened test scores they contain) for research, as many scholars have done. The challenge, however, is in negotiating data use agreements with each of the 50 states; without 50 separate data agreements, the use of student-level longitudinal data comes at the cost of full population coverage. Ideally, states might work together to create common systems for sharing de-identified individual educational records that would make it possible to conduct longitudinal student-level analyses with full population coverage; until that time, researchers will face a trade-off between using inferior data with full population coverage or more complete data in samples or subsets of the population.

Discussion, Part Two: The Heterogenity of Opportunity

As noted above, one of the advantages of having data on the full population of students, as opposed to a relatively small sample of students or districts, is that both general patterns and variation become clear. The analyses above demonstrate several key facts, some of which would not be evident without data of this kind.

First, there is enormous variation among districts in the extent of early learning opportunities available to children before third grade. These differences in opportunities are evident in the wide range of average third grade test scores. Not surprisingly, early opportunities are strongly associated with districts' socioeconomic characteristics; affluent families and districts are able to provide much greater opportunities than poor ones early in children's lives.

What may be surprising, however, is the extent of variation among districts in the kinds of opportunities they provide for students to learn from grades three to eight, and the fact that these growth opportunities are at best weakly correlated with early opportunities and with socioeconomic status. This is consistent, however, with other work showing that patterns of achievement do not correspond closely to patterns of test score growth (Hanselman and Fiel 2017). The empirical patterns evident presented above are most similar to the scenario described in Panel D of Figure 1 above: both early and middle childhood opportunities vary widely among school districts, but do not covary significantly.

It is tempting to think of growth rates in test scores as a rough measure of school district effectiveness. This is neither entirely inappropriate nor entirely accurate. The growth rates better isolate the contribution to learning due to experiences during the schooling years than do the grade 3 scores. Grade 3 average scores are likely much more strongly influenced by early childhood experiences than the growth rates. So the growth rates are certainly better as measures of educational opportunities from age 9 to 14 than are average test scores in a school district. But that does not mean they reflect only the contribution of schooling. Other characteristics of communities, including family resources, after school programs, and neighborhood conditions may all affect growth in test scores independent of schools' effects. Thus, some caution is warranted in interpreting the average growth rates as pure measures of school effectiveness. Nonetheless, relative to average test scores (at grade 3 or any grade), the growth rates are certainly closer to a measure of school effectiveness. Given that schooling plays a significant role

in children's lives from age 9 to 14 (at least in terms of time spent), it is not unreasonable to think that the growth measures carry some signal regarding school quality—and more signal than contained in simple average test score measures.

If we take the growth rates, then, as rough measures of school effectiveness, then neither socioeconomic conditions nor average test scores are very informative about school district effectiveness. Many districts with high average test scores have low growth rates, and vice versa. And many low-income districts have above average growth rates. This finding calls into question the use of average test scores as an accountability tool or a way of evaluating schools. Because average test scores, even in eighth grade, are only weakly correlated with growth rates, any system that rewards or sanctions schools or districts on the basis of their average scores will necessarily do so inappropriately in many cases (assuming that we wish to incentivize growth rates). And any information system that makes average test scores publicly available to parents in the hopes that a market for high test score districts will emerge and drive school improvement may instead simply create a market for high-SES districts, increasing economic segregation without improving school systems. To the extent that public information about school quality affects middle- and high-income families' decisions about where to live, information on growth rates might provide very different signals, perhaps leading to lower levels of economic residential and school segregation.

That is not to say the growth rates of the type I have calculated here–using repeated cross-sectional aggregated data—are ideal, but they almost certainly are better signals of the learning opportunities available in a school district than are average test scores. If we used measures like these as one part of an accountability system or a public information system, schools in the upper-left quadrant would be preferred (at least in grades 3-8) over districts in the lower-right quadrant. Future research might compare the growth measures I construct here with growth measures based on longitudinal student-level data. Such measures would be immune from the potential noise in my measures that arises

because of district in- and out-migration and/or grade retention.

The findings here also provide some insight into the issues raised in the opening of this paper. Are schools engines of opportunity or agents of inequality? The answer is perhaps more nuanced than the question implies. Some school districts seem to provide high opportunities for children from low-income families during elementary and middle school; others do not. That suggests our school systems (or other community institutions) have the potential to catalyze opportunity, but that potential is incompletely realized in many places. And although poverty is systematically associated with low opportunities to learn in early childhood, as evidenced by the consistently low average third grade test scores in low-income districts, poverty very clearly does not strictly determine the opportunities for children to learn in the middle grade years. That said, it is not clear from the patterns here that an effective school system alone can make up for low opportunities in early childhood. The large gaps in students' academic skills between low- and higher-SES districts are so large that even the highest growth rate in the country would be insufficient to close even half of the gap by eighth grade.

These patterns have implications both for education policy and for our understanding of the potentially equalizing role of schools. In terms of policy, they suggest that levels of student outcomes are a poor measure of school effectiveness. I am certainly not the first to say this, but the data from 11,000 school districts demonstrate the point very clearly. The findings also suggest that we could learn a great deal about reducing educational inequality from the low-SES communities with high growth rates. They provide, at a minimum, an existence proof of the possibility that even schools in high-poverty communities can be effective. Now the challenge is to learn what conditions make that possible and how we can foster the same conditions for children everywhere.

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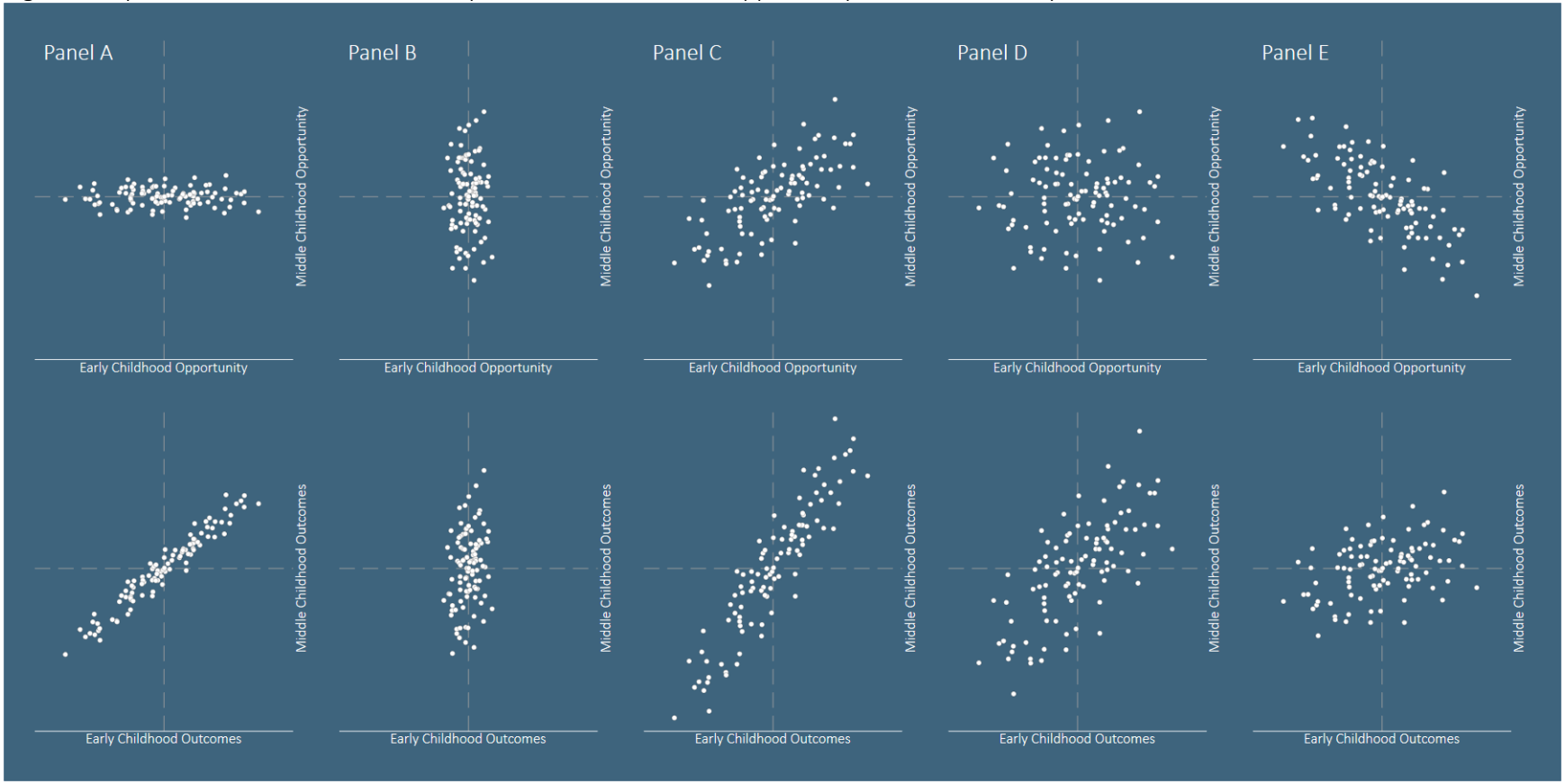
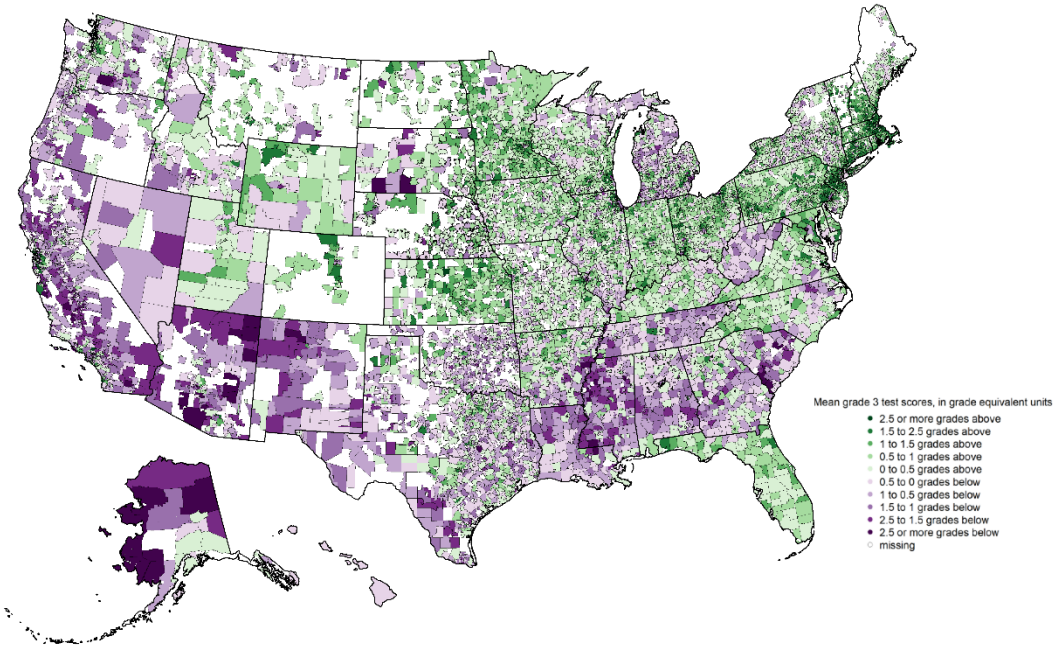
Figure 1: Stylized Associations Between Early and Middle Childhood Opportunity and Between Early and Middle Childhood Educational Outcomes



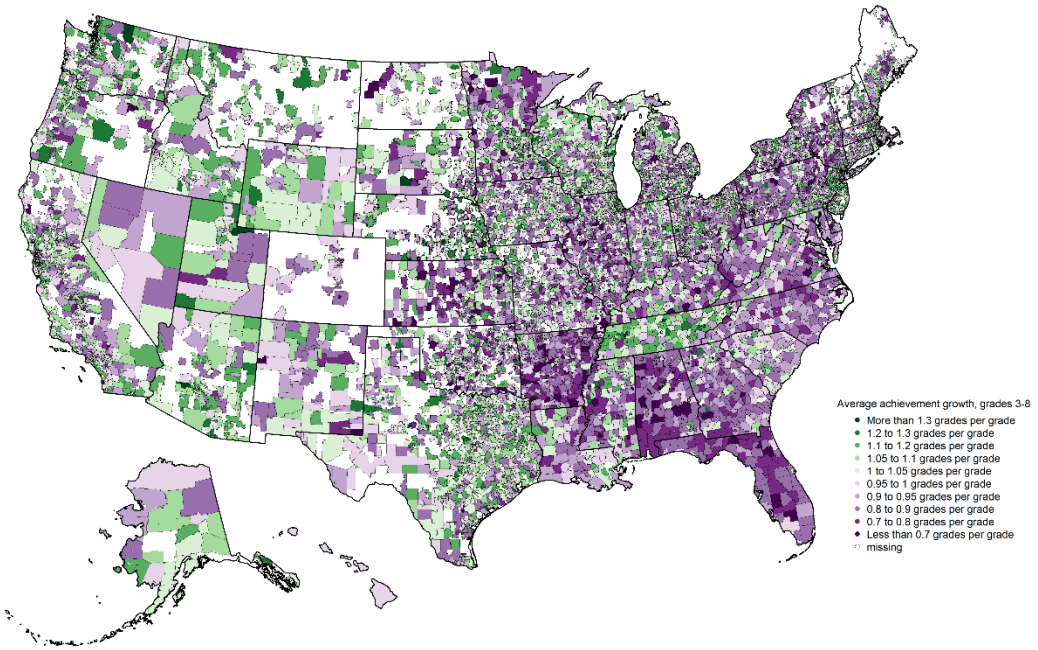
Note: In each panel, the top figure represents a stylized pattern of the distribution of early and middle childhood educational opportunities. The bottom panel represents the pattern of educational outcomes we would observe at the end of early and middle childhood. In panel A, for example, middle childhood educational opportunities vary little among communities. As a result outcomes at the end of middle childhood are highly correlated with outcomes at the end of early childhood: inequality neither grows nor shrinks. In Panel C, however, middle childhood opportunities are both variable and correlated with early childhood opportunities. As a result, inequality grows during middle childhood: middle childhood outcomes are more unequal then early childhood outcomes. The opposite pattern is shown in Panel E.

Figure 2:
Average Third Grade Test Scores (Math and Reading Averaged), US Public School Districts, 2009-2015



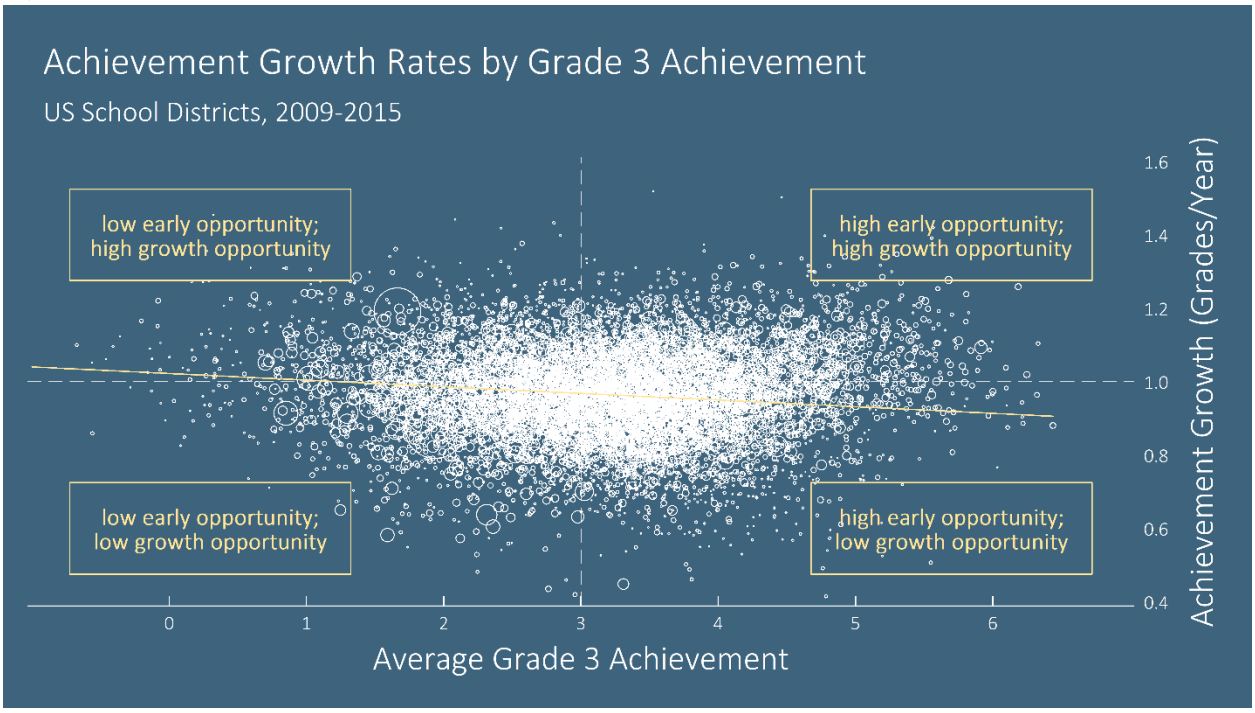
Source: Stanford Education Data Archive (Reardon et al. 2017a).

Figure 3:
Average Test Score Growth Rates (Math and Reading Averaged), US Public School Districts, 2009-2015



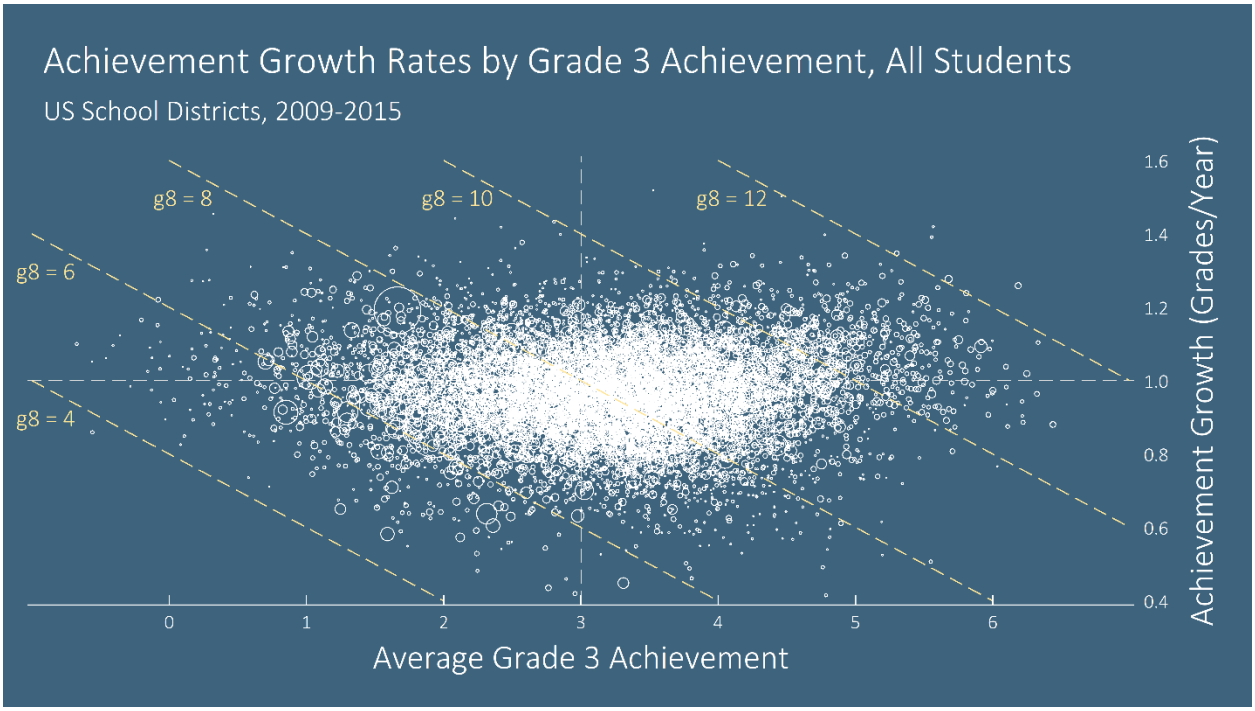
Source: Stanford Education Data Archive (Reardon et al. 2017a).

Figure 4:



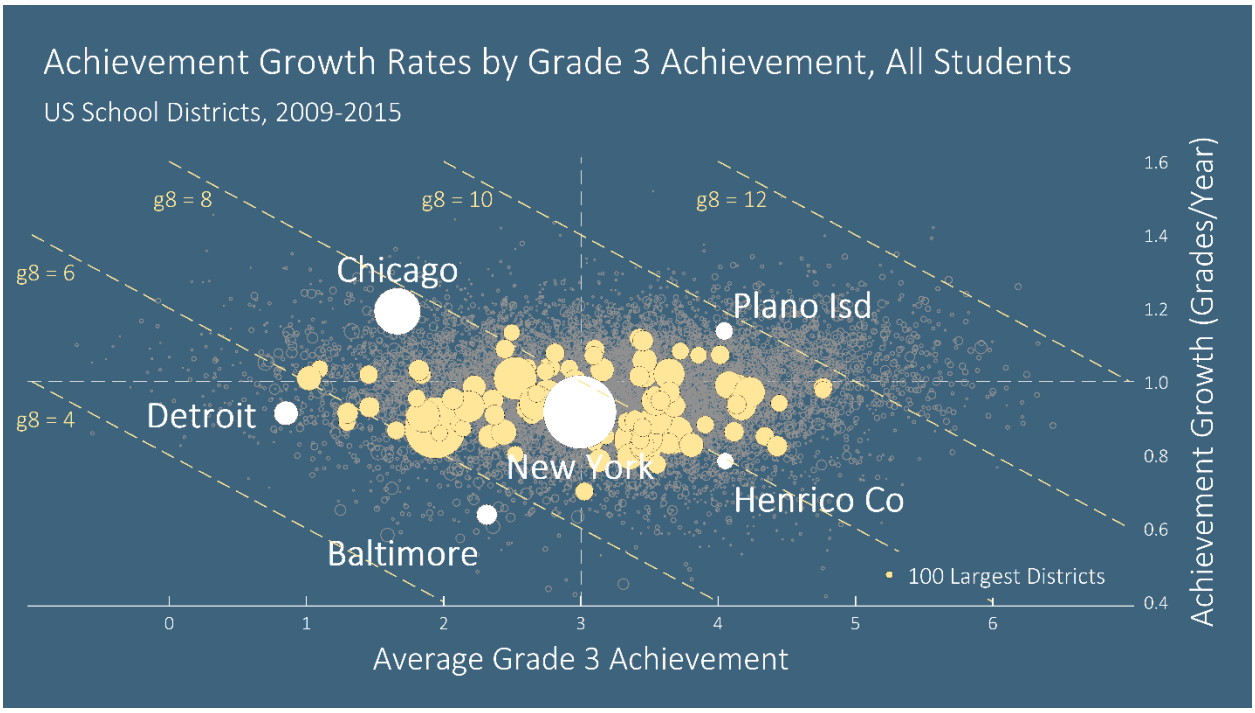
Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 5:



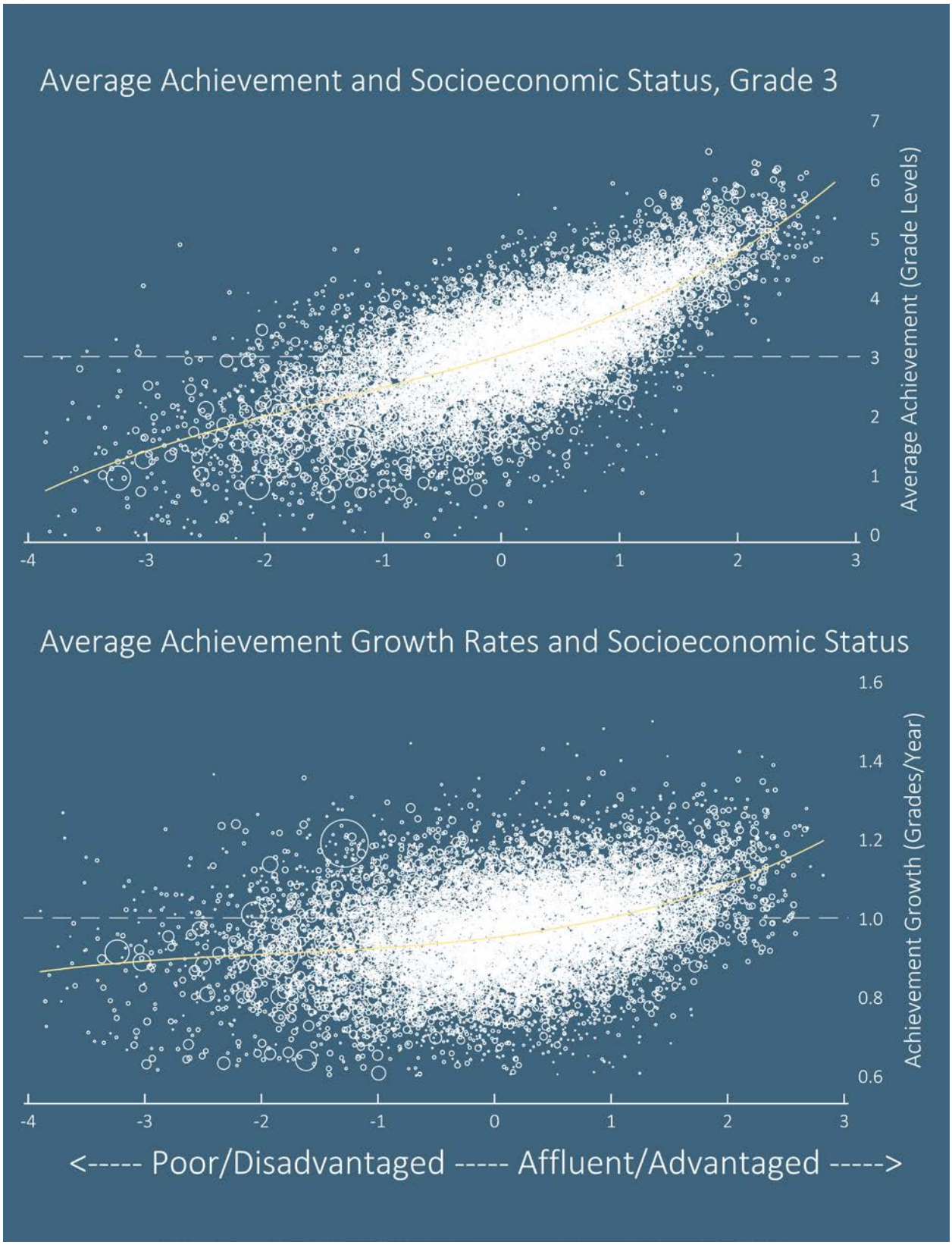
Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 6:



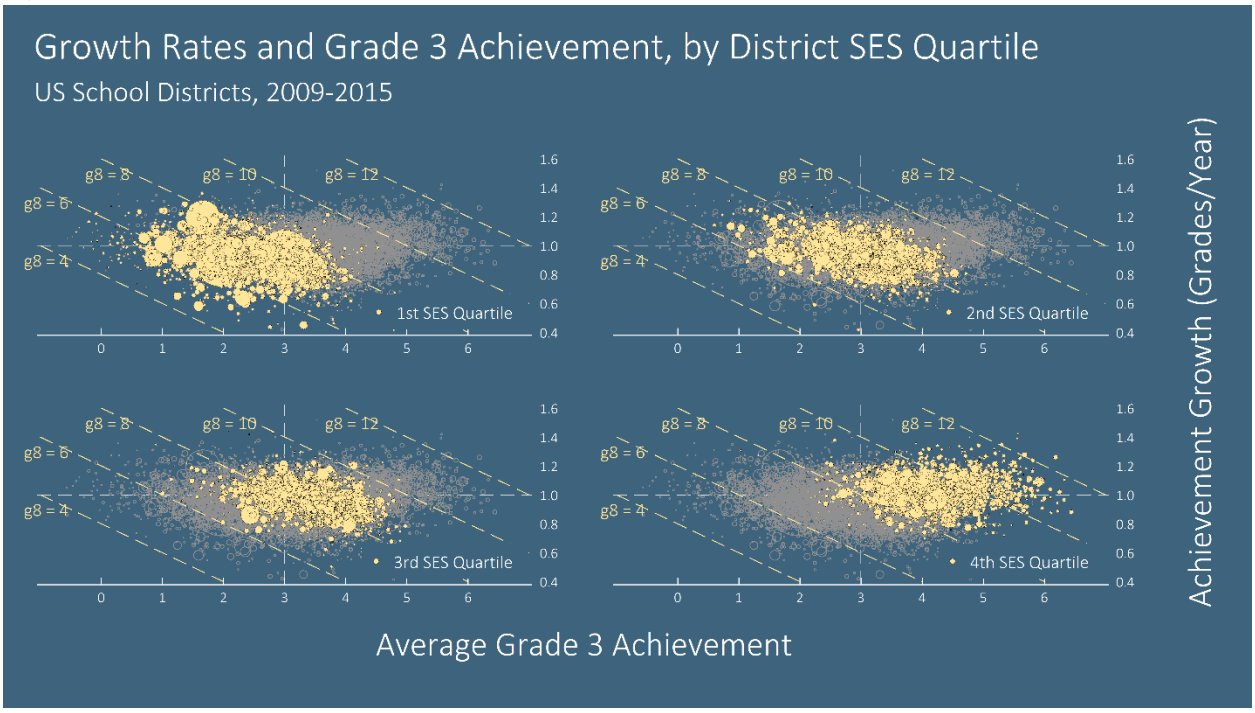
Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 7:



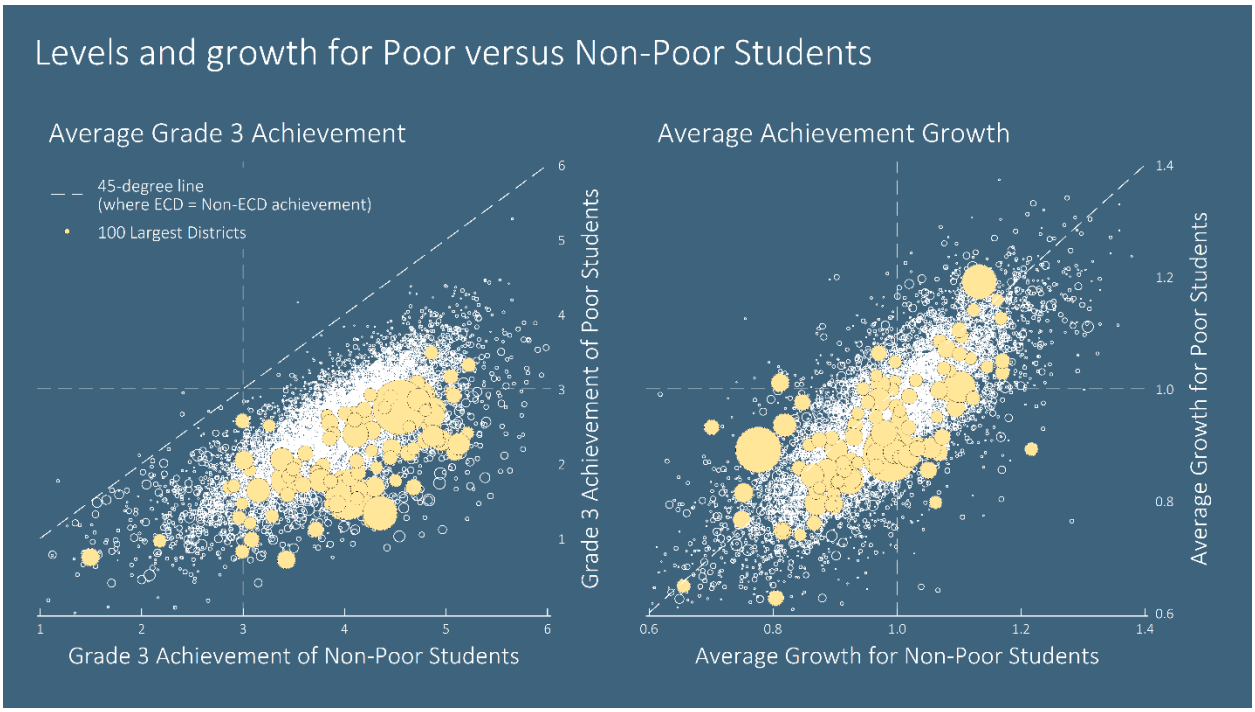
Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 8:



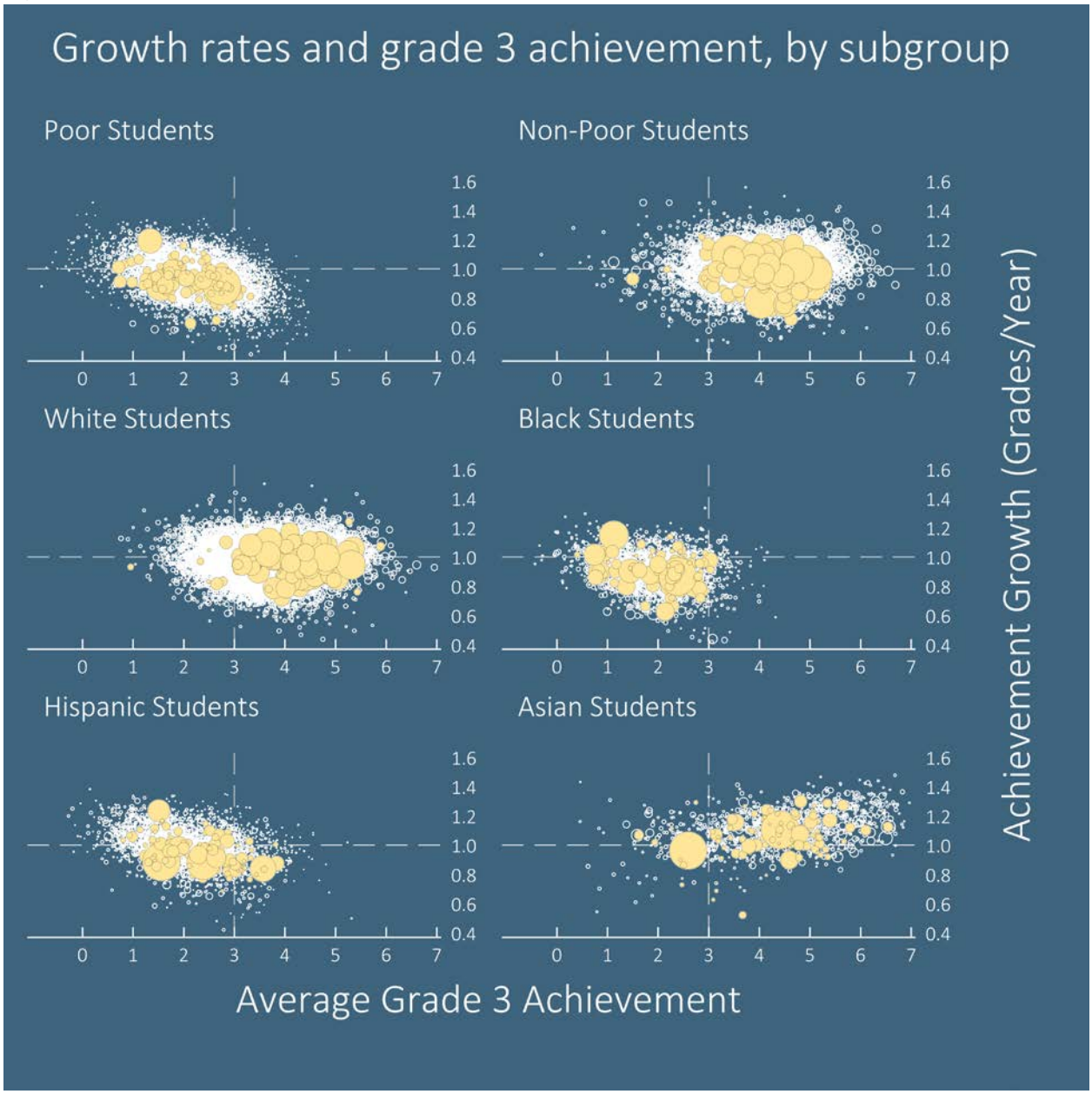
Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 9:



Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Figure 10:



Source: Author's tabulations, Stanford Education Data Archive (Reardon et al. 2017a).

Table 1:

Average Family Socioeconomic Characteristics, at Various District SES Composite Values

	SES Composite					
	-3	-2	-1	0	1	2
Median Family Income	$24,038	$31,026	$39,634	$53,029	$78,644	$136,804
% With BA or Higher	13.5%	14.9%	14.6%	18.3%	32.3%	62.4%
Poverty Rate	48.0%	37.6%	25.9%	14.7%	6.0%	1.6%
SNAP Eligibility Rate	50.0%	39.9%	27.6%	15.5%	5.6%	0.2%
Unemployment Rate	10.5%	8.0%	6.0%	4.5%	3.4%	2.6%
Single Parent Family Rate	51.9%	41.9%	31.7%	22.2%	14.6%	10.0%

Source: Fahle et al (2017), Table 6.

Table 2: Characteristics of the Joint Distribution of Grade 3 Test Scores and Grade 3-8 Growth Rates

	NAEP (Grade Eq.) Scale			Standardized Scale		
	Pooled	Math	ELA	Pooled	Math	ELA
Grade 3 Average						
Average	3.173	3.172	3.173	0.051	0.054	0.046
SD(Grade 3 Average)	0.976	0.919	1.084	0.341	0.361	0.337
Reliability(Grade 3 Average)	0.956	0.925	0.937	0.959	0.938	0.932
Growth, Grades 3-8						
Average	0.965	0.970	0.964	-0.008	-0.009	-0.005
SD(Growth, Grades 3-8)	0.135	0.175	0.123	0.044	0.055	0.040
Reliability(Growth, Grades 3-8)	0.859	0.843	0.754	0.854	0.822	0.749
Correlations						
Corr(Grade 3, Growth)	-0.130	0.002	-0.365	-0.245	-0.282	-0.241
Corr(Average Grades 3-8, Growth)	0.213	0.430	-0.086	0.079	0.100	0.057
Corr(Grade 8, Growth)	0.494	0.690	0.214	0.381	0.443	0.341
Corr(Grade 3, Grade 8)						
Corr(Grade 3 Math, Grade 3 Reading)		0.902			0.909	
Corr(Math Growth, Reading Growth)		0.661			0.760	
Predicted Average Scores by District Type						
Grade 3 Average Scores						
High Early/Average Growth Opportunity	4.149	4.091	4.257	0.392	0.415	0.383
Average Early/High Growth Opportunity	3.173	3.172	3.173	0.051	0.054	0.046
Difference	-0.976	-0.919	-1.084	-0.341	-0.361	-0.337
Grade 8 Average Scores						
High Early/Average Growth Opportunity	8.974	8.941	9.077	0.354	0.368	0.358
Average Early/High Growth Opportunity	8.673	8.895	8.610	0.233	0.280	0.221
Difference	-0.301	-0.045	-0.467	-0.121	-0.088	-0.137
Relative Magnitude of 1 SD of High Growth to 1 SD High Early Opportunity on Grade 8 Scores	0.692	0.950	0.569	0.645	0.757	0.593
N(Districts)	11,315	11,315	11,315	11,315	11,315	11,315

Source: Author's calculations, Stanford Education Data Archive (Reardon et al. 2017a).

Table 3: Characteristics of the Joint Distribution of Grade 3 Test Scores and Grade 3-8 Growth Rates, by Subgroup

	All	Poor	Non-Poor	White	Black	Hispanic	Asian	Male	Female
Grade 3 Average									
Average	3.173	2.351	3.803	3.535	1.933	2.177	4.286	3.088	3.274
SD(Grade 3 Average)	0.976	0.779	0.791	0.808	0.762	0.883	1.215	1.000	0.959
Reliability(Grade 3 Average)	0.956	0.901	0.913	0.931	0.899	0.881	0.899	0.943	0.941
Growth, Grades 3-8									
Average	0.965	0.942	0.985	0.967	0.912	0.992	1.110	0.932	0.998
SD(Growth, Grades 3-8)	0.135	0.134	0.133	0.131	0.131	0.134	0.144	0.137	0.129
Reliability(Growth, Grades 3-8)	0.859	0.809	0.811	0.831	0.796	0.770	0.719	0.829	0.819
Correlations									
Corr(Grade 3, Growth)	-0.130	-0.475	-0.167	-0.148	-0.298	-0.431	0.273	-0.089	-0.087
Corr(Average Grades 3-8, Growth)	0.213	-0.050	0.248	0.251	0.138	-0.057	0.508	0.245	0.242
Corr(Grade 8, Growth)	0.494	0.403	0.563	0.556	0.509	0.341	0.668	0.512	0.505
N(Districts)	11,315	9,735	10,180	10,662	3,077	4,102	1,789	10,327	10,233

Source: Author's calculations, Stanford Education Data Archive (Reardon et al. 2017a).

Appendix: Scale sensitivity of correlations between growth and status

The correlation between initial status (grade 3 test average test scores in our case) and growth (change in average scores from grade 3 to 8 here) is sensitive to the relative scales in which initial and final scores are measured. To see this, let $Y3$ and $Y8$ represent scores in grade 3 and 8, respectively. Let $\Delta = Y8 - Y3$ the change in scores. Let $\tau_3 = Var(Y3)$; $\tau_\Delta = Var(\Delta)$; and $C = Cov(Y3, \Delta)$. Note that the correlation of growth and initial status is then $r_{\Delta 3} = Corr(Y3, \Delta) = \frac{C}{\sqrt{\tau_3 \tau_\Delta}}$.

Now suppose we transform $Y8$ by a linear transformation, where $b > 0$:

$$Y8' = a + bY8.$$

The change as measured in this new metric is now:

$$\begin{aligned}\Delta' &= a + bY8 - Y3 \\ &= a + b\Delta + (b-1)Y3.\end{aligned}$$

The variance of changes in the new metric is:

$$\tau_{\Delta'} = b^2\tau_\Delta + (b-1)^2\tau_3 + 2b(b-1)C.$$

And now the correlation of $Y3$ and Δ' will be

$$\begin{aligned}r' &= \frac{Cov(Y3, \Delta')}{\sqrt{\tau_3 \tau_{\Delta'}}} \\ &= \frac{bC + (b-1)\tau_3}{\sqrt{\tau_3(b^2\tau_\Delta + (b-1)^2\tau_3 + 2b(b-1)C)}} \\ &= \frac{\left(b\sqrt{\tau_\Delta} + (b-1)\sqrt{\tau_3}\right) + (r-1)b\sqrt{\tau_\Delta}}{\left[\left(b\sqrt{\tau_\Delta} + (b-1)\sqrt{\tau_3}\right)^2 + 2b(b-1)(r-1)\sqrt{\tau_3\tau_\Delta}\right]^{\frac{1}{2}}}.\end{aligned}$$

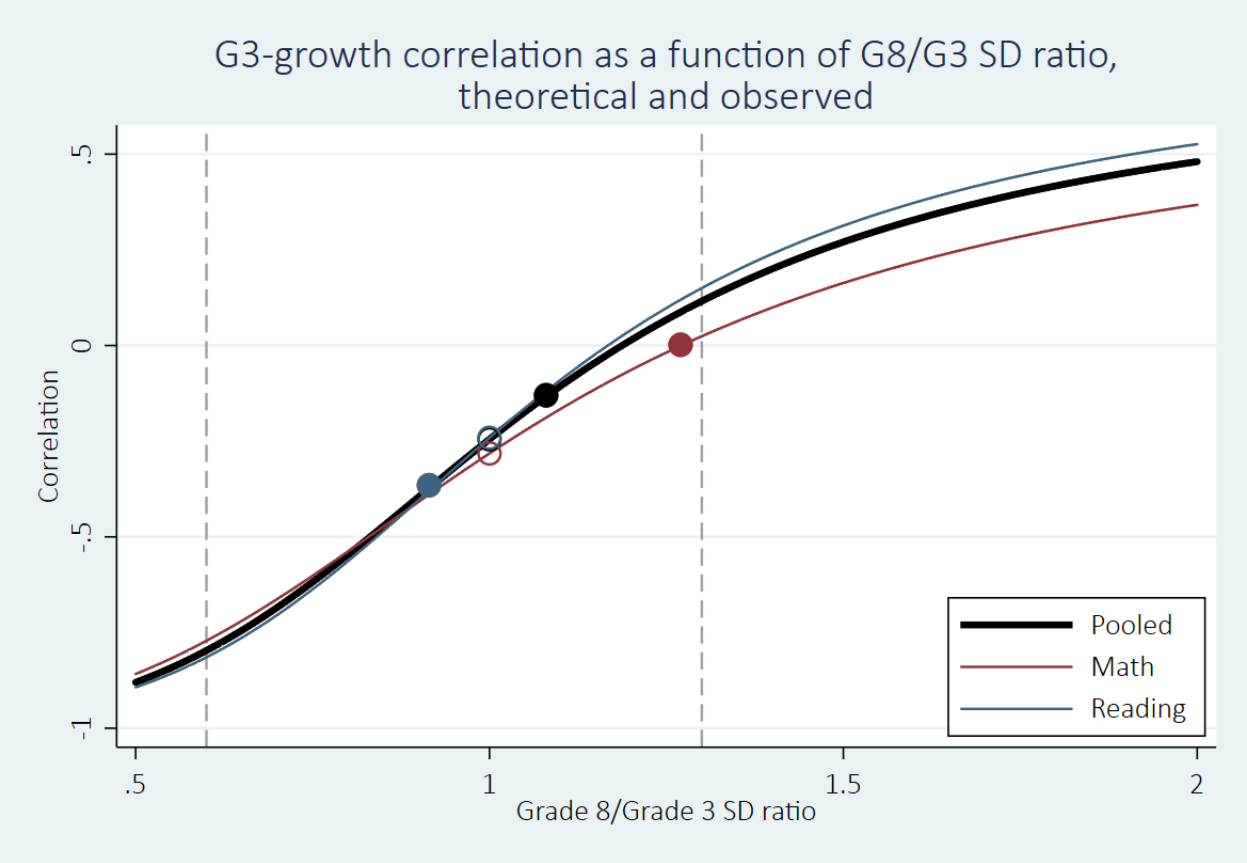
(A1)

Now, given τ_3, τ_Δ, and C (or r), r' is a continuous, monotonically increasing function of b. Note that

$$\lim_{b\to\infty} r' = Corr(Y3, Y8)$$

$$\lim_{b\to 0} r' = -1.$$

If we take the estimated values of τ_3, τ_Δ, and C estimated from Model 1 using the standardized test scale (the scale in which $\tau_3 = \tau_8$), we can plot the equation (A1) as a function of b.



Source: Author's calculations.

The red line, for example, displays the correlation between math average third grade scores and growth rates as a function of b. In the standardized scale (corresponding to $b = 1$ on the figure), the estimated correlation is -0.282 (as shown by the hollow red circle). In the NAEP scale, the estimated correlation is 0.002, indicated by the solid red dot. This occurs at a value of about $b = 1.25$, which is very close to the ratio of the eighth grade NAEP math standard deviation to the 4th grade standard deviation. In other words, NAEP scale has a value of roughly $b = 1.25$ in math (and a value of $b = 0.94$ in reading).

In order to produce a correlation of $r' > 0.25$, we would need $b > 1.5$ in reading and $b > 1.7$ in math. In other words, if the 8th grade metric were stretched by a factor of 1.7 or 1.5 in reading or math, respectively, the estimated correlation would be positive 0.25 rather than -0.25 – still a low correlation

but with the opposite sign as we observe in the standardized scale. Is a factor of $b > 1.5$ plausible?

One way to assess this is to examine other vertically scaled tests. Dadey and Briggs (2012) examine 16 vertically scaled tests used in state assessment programs. For these tests, the value of b—the ratio of the 8th grade standard deviation of scores to the 3rd grade standard deviation—ranges from 0.6 to almost 1.3 (though most of the reading ratios are between 0.8 and 1.0; most of the math ratios are between 0.9 and 1.1). Bloom et al (2008) report standard deviations for 7 vertically equated reading tests; the grade 8 to grade 3 standard deviation ratios in those tests range from 0.87 to 1.04. Of the 23 vertically scaled assessments for which data are available, none have $b > 1.3$. The vertical grey dashed lines in the Figure show the range of values of b reported by Dadey and Briggs (2012) and Bloom et al (2008). The possible correlations these values of b would produce in the SEDA data range from -0.80 to $+0.15$. This suggests that there is no plausible vertical scale that would yield a moderate or high positive correlation between grade three test scores and growth rates.